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The Anaerobic Digester as a Bistable System: A History-Dependent Climate Credit and Its Early-Warning Signal

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> *Preprint, working draft (English version). Code and data are reproducible and openly deposited (Zenodo DOI at the first stable version). Scientific companion to the ChaoticLCA preprint (Piens 2026, Zenodo 10.5281/zenodo.20086772). No competing interests declared.*
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> *This article belongs to the research program "Waste as dynamical systems" (Piens, in preparation), of which it is the demonstrative case study. It can be read on its own; the theoretical apparatus (implicit axioms S1 to S4, theorems T1 to T4) is developed in the companion Manifesto.*

Abstract

Context. Life cycle assessment (LCA) credits anaerobic digestion with a fixed climate benefit: the methane produced avoids natural gas, and this credit is carried, in environmental product declarations, by a module whose value is assumed constant (module D of the EN 15804 standard). Yet anaerobic digestion is a microbial ecosystem whose dynamics are known to be bistable: at a given organic load, the same reactor can occupy a healthy methanogenic state or an acidified state, and switch from one to the other.

Objective. We examine the consequence of this bistability, established in the literature, for the climate credit attributed to the process in LCA, and we specify the contribution of a dynamic early-warning signal, complementary to the threshold-based chemical indicators already used in operation.

Methods. We use the reduced model AM2 (Bernard et al., 2001), identified on a real 1 m^3 digester treating wine-distillery effluent. We compute the equilibrium branches along the dilution rate by continuation, locate the saddle-node analytically, translate each branch into a greenhouse-gas balance, and estimate the return time to equilibrium by diagonalizing the Jacobian along the healthy branch.

Results. On the identified parameter set, the saddle-node is located at a dilution rate `D_fold ≈ 0.587 d⁻¹`. In the bistable window, the same digester, at identical load, is a sink or a source of greenhouse gas depending on its initial state alone, that is, on its history. As the saddle-node is approached, the return time to equilibrium passes through a minimum of about four days then diverges, which constitutes a measurable early-warning signal of the switch.

Conclusion. For a bistable process, the climate credit attributed to the functional unit is not only uncertain, it is ill-posed in the bistability window, because it depends on the trajectory and not on the load alone. We propose that the LCA of such processes report the regime, and not the quantity alone, and that critical slowing down, a dynamic reading of the chemical indicators already monitored,

complete the surveillance arsenal. We present this work as the application, to LCA, of established results of dynamical-systems theory and the ecology of critical transitions, and not as a new mathematical result; its novelty lies in the bridge to credit accounting.

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1. Introduction

Anaerobic digestion occupies a central place in the environmental balance sheets of organic-waste management and bioenergy. In life cycle assessment, its climate benefit is generally accounted for as a credit: the biomethane produced substitutes for fossil natural gas, and the corresponding avoided emission is entered. This credit is not incidental. It is often what tips the scales in favor of methanization in comparisons of treatment routes, and it underpins part of the support policies for bioenergy. Its correctness therefore engages real investment and orientation decisions, and the question of whether it is well defined is not merely methodological. Within the normative framework of environmental product declarations for construction products and other sectors, this benefit is carried by a module dedicated to the avoided material or energy, whose value enters the calculation as a constant attached to the functional unit (EN 15804 standard).

This accounting implicitly assumes that the process has a well-defined operating point, around which methane production, hence the credit, varies regularly with the load. But a digester is not a fixed-yield box, it is a population: billions of micro-organisms that one feeds, that digest, and that, like any living population, can thrive or collapse. From the dynamical point of view, it is a microbial ecosystem of two coupled populations: acidogenic bacteria produce volatile fatty acids, which methanogenic archaea consume into methane. This second step inhibits itself through its own substrate. From this inhibition follows a well-documented property: the system is *bistable*. At one and the same organic load, two stable states can coexist, a productive methanogenic state and a non-productive acidified state, and the reactor can switch from one to the other under perturbation. The bistability and saddle-node structure of reduced anaerobic-digestion models have been rigorously established (Meadows et al., 2019), and the risk of switching toward biomass washout through acid accumulation is a classical object of process control.

The gap we address is the following: the LCA accounting of the climate credit ignores this bistability, although it changes its very nature. Our contribution holds in one paragraph. On the AM2 model identified on a real reactor, we show that, in the bistability window, the greenhouse-gas balance of the process changes sign depending on the state occupied: at identical load, the same digester is a sink or a source, depending on its history and not on the load. It follows that the credit attributed to the functional unit is not a well-defined quantity in this window, a direct consequence of the bistability, the saddle-node being only its boundary, the washout. We then show that critical slowing down, the dynamic signature of the approach to a bifurcation, provides a measurable early-warning signal of the switch. We announce the three quantified results: a saddle-node at $D_{fold} \approx 0.587\ d^{-1}$, a sign change of the climate balance in the bistable window, and a divergence of the return time to equilibrium as the fold is approached.

We stress at the outset the nature of the contribution. Bistability and critical slowing down are established results of dynamical-systems theory (Strogatz, 2018) and the ecology of critical transitions (Scheffer et al., 2009). The novelty claimed here is not mathematical: it is the matching of these known properties with the accounting of the climate credit in LCA, and the methodological conclusion that follows.

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2. State of the art

Modeling and instability of anaerobic digestion. Two families of models coexist. The ADM1 model (Batstone et al., 2002) is a detailed reference model with a large number of states. The AM2, or AMOCO, model (Bernard et al., 2001) is a reduced model with two steps and four state variables, designed for control and monitoring, and identified on a real pilot digester. The loss of stability of anaerobic digestion through overload, which leads to the accumulation of volatile fatty acids then to the washout of methanogenic biomass, is a phenomenon observed in operation and a central object of process control. The mathematical analysis of these reduced models has established the existence of a bistability and a saddle-node bifurcation, as well as the convergence of every trajectory toward an equilibrium point, with no limit cycle, including in the bistable regime (Meadows et al., 2019). The operating diagram, which maps the regimes as a function of the operating parameters, is now well characterized for these two-step models. In practice, the loss of stability manifests as the accumulation of volatile fatty acids and the fall in buffering capacity, which operators monitor by means of dedicated state indicators (Boe et al., 2010); keeping the reactor at a distance from washout is an explicit objective of the control strategies of anaerobic digestion. The phenomenon we study is therefore neither hypothetical nor marginal: it is the failure mode that operation seeks precisely to avoid.

Critical transitions and early-warning signals. In ecology and beyond, the approach to a saddle-node bifurcation is accompanied by a *critical slowing down*: the dominant eigenvalue of the linearized system tends toward zero, and the return time to equilibrium after a perturbation increases without bound as the fold is approached. This slowing down leaves traces in the ordinary fluctuations of the system, in the form of a rise in autocorrelation and variance, which one knows how to detect on real series without knowing the underlying model (Scheffer et al., 2001; Scheffer, 2009). This genericity is the whole of their operational interest; their limit is known too: they signal the approach of a transition, not its date, and certain classes of bifurcation leave them mute. The statistical methods for detecting these signals on real series, including the handling of false positives and seasonality, are today standardized (Dakos et al., 2012); we detail their reading and the protocol in section 5.3.

Early warning in anaerobic digestion: a precedence to acknowledge. Let us be clear on this point, for it decides the nature of our contribution. The early detection of digester failure is not virgin ground; it is even a well-furnished field. A dedicated body of work has identified effective indicators, foremost among them the ratio of intermediate to bicarbonate alkalinity (IA/BA) and the volatile-fatty-acid concentration, which offer warning times of the order of five to eight days on real digesters, demonstrated on food waste (Wu et al., 2014), on vegetable waste (Li et al., 2017), and up to the dry route at full scale. We therefore make no claim to introduce early warning into anaerobic digestion. But these indicators are, for the most part, chemical and threshold-based: a quantity crosses a critical value, and the alarm triggers. Critical slowing down is of another kind. It reads not the level of a quantity, but the dynamics of its fluctuations, the rise of its variance and autocorrelation as the saddle-node is approached; and it has been observed directly, on a real microbial population whose dilution rate was varied up to the tipping point (Dai et al., 2012), that is, in a system and with a control parameter closely analogous to those of a digester. Our contribution, on this plane, is therefore not to propose an additional alarm. It is twofold: to provide the dynamic underpinning, the saddle-node and critical slowing down, of indicators the field already uses empirically, and to connect this underpinning to a consequence that, for its part, is new, the accounting of the climate credit in life cycle assessment.

LCA of anaerobic digestion and the treatment of uncertainty. The climate benefit of anaerobic digestion is accounted for in LCA as an avoided emission, carried by a fixed-value module (EN 15804). The treatment of uncertainty in LCA has been the subject of important developments beyond the simple log-normal margin: Monte-Carlo propagation, accounting for correlations between parameters, global sensitivity analysis (Heijungs & Suh, 2002; Groen et al., 2014). These methods are powerful, but they share an assumption we will discuss: they propagate a *parametric* uncertainty through a *fixed* model structure, and produce a unimodal output distribution. They are not designed to represent a *bimodal and history-dependent* output, which is the signature of a bistable process.

Positioning. The coupling of system dynamics to life cycle assessment is an already constituted field (McAvoy et al., 2021). But its object is the *temporal* trajectory of a product system, the renewal of a fleet, the diffusion of a technology, the evolution of an energy mix. Our object is distinct: it is not the evolution of a system in time, but the *multistability* of a process at fixed conditions, that is, the coexistence, at the same operating point, of two stable states that history sets apart. To our knowledge, the consequence of this process bistability for the accounting of the climate credit in LCA has not been treated. Our contribution sits at this intersection: it is one of framing and tooling, not of new dynamics. We take an established result (the bistability of AM2) and draw from it a conclusion for a practice (credit accounting), then propose an operational indicator (critical slowing down).

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3. Methods

3.1 Model

We use the AM2 model (Bernard et al., 2001). It describes two populations, acidogenic `X1` and methanogenic `X2`, and two substrates, the organic substrate `S1` and the volatile fatty acids `S2`, in a reactor of dilution rate `D`:

```
dX1/dt = (μ1(S1) − α D) X1
dX2/dt = (μ2(S2) − α D) X2
dS1/dt = D (S1in − S1) − k1 μ1(S1) X1
dS2/dt = D (S2in − S2) + k2 μ1(S1) X1 − k3 μ2(S2) X2
```

Acidogenic growth follows Monod kinetics and methanogenic growth follows Haldane kinetics, inhibited by the substrate:

```
μ1(S1) = m1 S1 / (S1 + KS1)
μ2(S2) = m2 S2 / (KS2 + S2 + S2²/KI2)
```

The coefficient `α ∈ [0, 1]` represents the fraction of biomass subject to washout (biomass retention). The methane flow rate is `qM = k6 μ2(S2) X2`. The parameter values, from the identification on a real reactor (Bernard et al., 2001), are given in Table 1. We place the inhibition coefficient `KI2` in the inhibited range that produces bistability, and we test the robustness of this choice in section 3.5.

Table 1. Parameters of the AM2 model.

Parameter	Value	Unit	Role
m1	1.20	d^{-1}	max acidogenic growth rate
KS1	7.1	$g \cdot L^{-1}$	half-saturation constant (Monod)
m2	0.74	d^{-1}	max methanogenic growth rate
KS2	9.28	$mmol \cdot L^{-1}$	half-saturation constant (Haldane)
KI2	16	$mmol \cdot L^{-1}$	inhibition constant (Haldane)
α	0.5	–	fraction of biomass washed out
k1	42.14	–	substrate/acidogen yield
k2	116.5	–	acid production by acidogenesis
k3	268.0	–	acid consumption by methanogenesis
k6	453.0	–	methane yield
S1in	15.0	$g \cdot L^{-1}$	incoming organic substrate
S2in	80.0	$mmol \cdot L^{-1}$	incoming fatty acids

3.2 Equilibrium branches and saddle-node

For each value of D, we compute the stationary states by integration to equilibrium (implicit solver, tolerances $rtol = 10^{-9}$, $atol = 10^{-11}$), from two initial conditions: a healthy inoculum (methanogenic biomass present) and an acidified inoculum (methanogens collapsed). These two seeds select the healthy branch and the acidified branch respectively. We follow each branch by continuation in D.

The saddle-node is located analytically. The Haldane kinetics μ2(S2) reaches its maximum

```
μ2,max = m2 / (1 + 2 √(KS2/KI2))
```

at $S2^* = \sqrt{KS2\ KI2}$. The methanogenic state exists only as long as $\alpha\ D < \mu_{2,max}$, that is

```
D < D_fold = μ2,max / α.
```

Beyond D_fold, no stationary methanogenic state exists and washout becomes global. For the parameters of Table 1, $S2^* \approx 12.2\ mmol \cdot L^{-1}$, $\mu_{2,max} \approx 0.293\ d^{-1}$, and $D_{fold} \approx 0.587\ d^{-1}$.

3.3 Translation into a greenhouse-gas balance

The methane flow rate qM at equilibrium, on each branch, is converted into a climate credit by substitution for natural gas. Against this credit we set a fixed operating load B, which represents the heating and mixing energy of the reactor. This load does not depend on the biological state: it persists when the reactor acidifies and ceases to produce. The net balance is

```
GWP_net(D, branch) = B − γ qM(D, branch),
```

where γ converts the methane produced into an avoided emission. On the healthy branch, qM is high and the balance is negative (sink). On the acidified branch, $qM \approx 0$ and the balance reduces to B, positive (source). We set B to a parasitic energy demand on the order of 10 to 30 % of the gross energy produced on the healthy branch, an order of magnitude documented for mesophilic digestion, where the heating of the digestate dominates and mixing counts for a few percent. This value is declared and illustrative; it fixes the *magnitude* of the balance, not its *sign*. This is a distinction we will return to, for it decides what, in this result, is trivial and what is not.

3.4 Critical slowing down

Along the healthy branch, we follow the stationary state by continuation in D, using the equilibrium obtained at the previous step as the seed of the next, which makes it possible to stay on the branch up to the immediate neighborhood of the fold. At each point, we evaluate the 4×4 Jacobian matrix of the system by centered finite differences, then compute its eigenvalues. The return time to equilibrium after a small perturbation is

```
τ = 1 / |Re(λ_dom)|,
```

where λ_dom is the eigenvalue of least-negative real part, that of the slowest mode. As a saddle-node is approached, $Re(\lambda_{dom}) \to 0$ and $\tau \to \infty$. Far from the fold, the slowest mode may be another mode of the system, whose characteristic time decreases as the reactor is loaded; only in the neighborhood of the saddle-node does the vanishing mode become the slowest and dominate. This results in a non-monotonic form of τ, first decreasing then diverging, from which we draw precisely the warning signal in section 4.3.

3.5 Robustness and reproducibility

We sweep the inhibition coefficient KI2 from 8 to 40 mmol·L^{-1} to check that the bistability window is not an artifact of a particular setting. The whole of the computations is deterministic and reproducible; the code and figures are openly deposited (Zenodo DOI at the first stable version).

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4. Results

4.1 Bistability and saddle-node

On the identified parameter set, the healthy methanogenic branch and the acidified branch coexist over a wide load window, up to the saddle-node located analytically at $D_{fold} \approx 0.587\ d^{-1}$ (section 3.2). The methanogenic branch exists up to this fold, and the two states remain clearly distinct over a load window from about 0.19 to 0.52 d^{-1}; beyond, healthy production declines until it dies out at the fold. Below this fold, the state occupied by the reactor is not determined by the load D alone: it depends on the seed, that is, on the history of the reactor. The bistability window resists the sweep of the inhibition coefficient KI2 from 8 to 40 mmol·L^{-1}: it is not an artifact of a single setting, but a property of the inhibited regime.

4.2 Sign change of the climate balance

Figure 1 translates the two branches into a greenhouse-gas balance. On the healthy branch, sustained methane production gives a negative net balance: the digester is a sink. On the acidified branch, at the same load, production collapses while the operating load persists, and the net balance becomes positive: the same digester is a source. Throughout the bistability window, the sign of the climate balance attributed to the functional unit therefore depends on the trajectory followed, and not on the declared load.

One must here guard against a false depth. The sign switch, taken by itself, is in no way surprising: a reactor that no longer produces but that one keeps heating emits while avoiding nothing, and the arithmetic of the balance says so at once. The non-trivial result is not there. It is in the fact, established in the previous section, that this non-productive state is *stable* at a load where the productive state is just as stable. It is this coexistence, and not the computation of the sign, that makes the balance depend on the history of the reactor rather than on the declared quantity. Bistability is the result; the sign change is only its accounting translation.

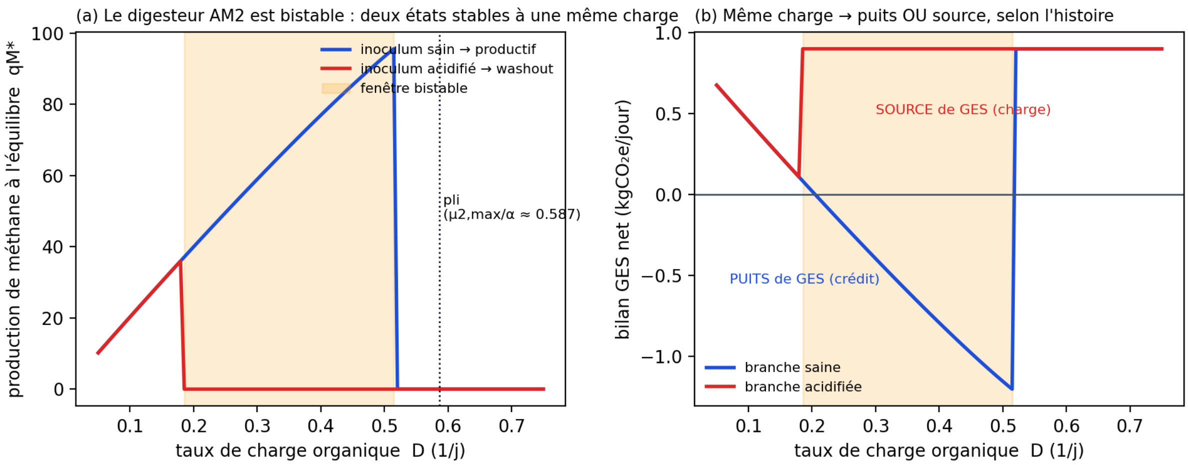

Figure 1. The anaerobic digester (identified AM2 model, Table 1) is bistable. (a) Methane production at equilibrium as a function of the organic load rate D, on the healthy branch (methanogenic inoculum) and on the acidified branch (acidified inoculum); the analytic fold ($\mu_{2,max}/\alpha \approx 0.587$ d^{-1}) marks the disappearance of the healthy state. (b) Net greenhouse-gas balance of the two branches: at identical load, the same reactor is a sink on the healthy branch and a source on the acidified branch. The magnitude depends on the assumed fixed operating load; the sign change does not.

4.3 Critical slowing down as a warning signal

Figure 2 shows, along the healthy branch, the slowest mode and the return time τ. As the reactor is loaded, τ first decreases, passes through a minimum of about four days near $D \approx 0.54$ d^{-1}, then diverges as the saddle-node $D_{fold} \approx 0.587$ d^{-1} is approached, rising beyond nine days. Table 2 gives a few values.

Table 2. Slowest mode and return time along the healthy branch.

D (d^{-1})	0.30	0.45	0.52	0.54	0.585
$Re(\lambda_{dom})$ (d^{-1})	−0.144	−0.208	−0.235	−0.241	−0.105
τ (d)	7.0	4.8	4.3	4.1	9.5

The minimum of τ near $D \approx 0.54$ d^{-1} then its clear rise at $D = 0.585$ d^{-1} materialize the trend reversal. It is not the absolute level of τ that constitutes the signal, but its reversal of trend: the moment when, even as the load keeps increasing, the reactor starts again to react more and more slowly to perturbations. This reversal precedes global washout, and it is measurable indirectly through the rise in autocorrelation and variance of the volatile fatty acids, without it being necessary to know the model.

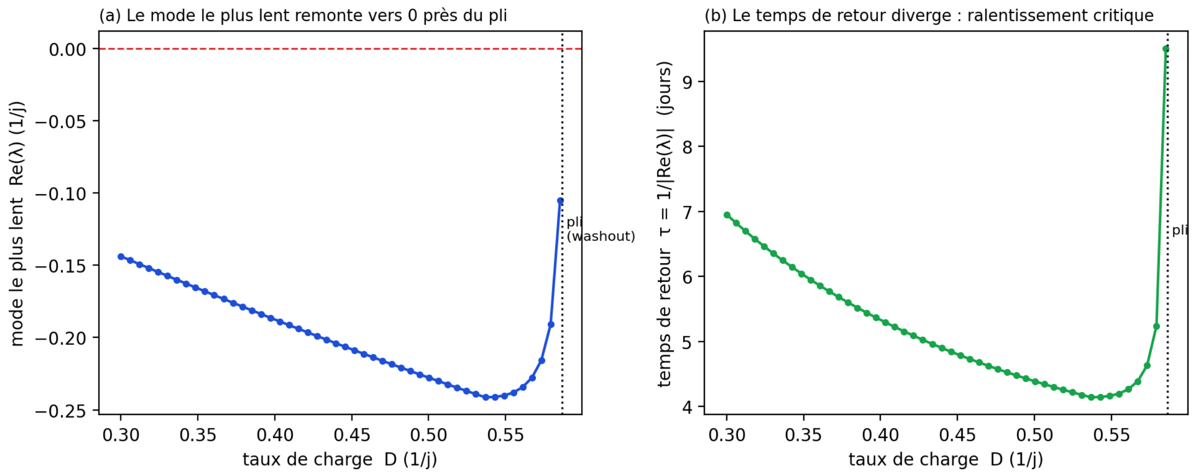

Figure 2. Critical slowing down along the healthy branch of the AM2 model. (a) Real part of the dominant eigenvalue (slowest mode) as a function of the load rate D; it rises toward zero as the saddle-node is approached. (b) Return time to equilibrium $\tau = 1/|\mathrm{Re}(\lambda)|$; after a minimum of about four days near $D \approx 0.54\ d^{-1}$, it diverges as the fold is approached ($D_{fold} \approx 0.587\ d^{-1}$). The trend reversal of τ announces the switch before it occurs.

4.4 Representativeness

It will be objected, and this is the most serious objection, that operators precisely avoid the neighborhood of the fold, and that the usual credit rests on yields measured on the healthy branch, far from the instability. The window in which our result bites would then be too narrow to matter. We take this objection seriously, and we answer it with three situations, none of which is marginal. First, the economics push against prudence: high-volumetric-load digestion, sought to maximize the methane flow per unit volume and hence the credit, deliberately brings the reactor closer to the inhibited regime where bistability appears. Second, and above all, the acidified state is not an abstraction: it is a real and frequent failure state, the very one that the whole early-warning literature exists to prevent, and a digester that has switched remains, during its recovery, a credited asset that produces little while consuming energy. Bistability moreover explains why this switch is tenacious: the acidified state being stable, one does not leave it merely by reducing the load. Finally, the considerable dispersion of yields observed between comparable installations is, in part, compatible with a branch effect, without one being able today to make the exact share. We do not have a quantitative census of the fraction of the fleet operating near the fold or in an acidified state; establishing this census, by re-reading operating series through the prism of regime, is one of the direct sequels this work calls for. As it stands, the phenomenon is binding not for every digester, but for those that are pushed, and for those that have switched, which does not make a negligible exception.

5. Discussion

5.1 An ill-posed credit, not merely uncertain

Below the saddle-node, the stationary output of the process is not a single-valued function of the load: at fixed D, two stable branches coexist, the healthy and the acidified one, separated by an unstable branch, and the one the reactor occupies depends on its history. This is the signature of an S-shaped (folded) equilibrium curve, whose saddle-node $\dot{x} = r - (x - x^*)^2$ marks the fold: the edge where the healthy branch is destroyed, the washout (Strogatz, 2018). The consequence for LCA is that a single climate credit attached to the functional unit is not, in this window, a poorly measured quantity: it is an *ill-posed* quantity. An average between the two branches describes no physical state of the reactor, since it holds to one or the other, never between the two. To enter a single number amounts to tacitly choosing one of the two possible histories of the process. On the accounting plane, this suggests a prudent rule: in the bistability window, and for lack of proof of robustness, the credit attributed should reflect the unfavorable branch rather than an average, the full benefit being claimed only for an operation whose distance to the fold has been verified by the monitoring described in section 5.3.

It is important to distinguish this result clearly from another, which resembles it but is separate. Our argument bears on the bistability of the *bioreactor dynamics*, that is, on the physical process. It does not bear on the propagation of inventory-data uncertainty through a product system, which is a distinct question. The first concerns the equations of the process; the second, the structure of an inventory and the quality of its data.

5.2 Why advanced uncertainty treatment does not suffice

It might be objected that the modern arsenal of uncertainty quantification in LCA, Monte-Carlo propagation, accounting for correlations between parameters, global sensitivity analysis (Groen et al., 2014), suffices to capture the phenomenon. It does not, and the reason is structural. These methods propagate a *parametric* uncertainty through a model structure held to be *fixed*: they draw parameters from their distributions, pass them through a given model, and produce an output distribution. Whatever its sophistication, that distribution remains that of a response assumed single-valued to dispersed inputs; it is unimodal by construction. Bistability, for its part, does not come from the dispersion of the inputs: it comes from the fact that the model itself possesses, for one and the same input, two stable states. The relevant output is then bimodal and history-dependent, which no parametric propagation through a fixed structure can generate. The flaw is therefore not a flaw of precision one corrects by refining the propagation; it is a flaw of form. To see it, it suffices to imagine a perfect Monte-Carlo analysis of the model parameters, run from a healthy inoculum: it would produce a narrow, unimodal distribution around the healthy branch, and would entirely miss the very existence of the acidified branch, which yet a simple perturbation of the seed suffices to reach. To recognize bistability is therefore not a refinement of uncertainty analysis, it is a prerequisite to it: one must first know which branch one is on, and at what distance from the fold, before propagating anything at all.

5.3 Critical slowing down as an operational indicator

The previous result is, in itself, bad news: there exists a window where the credit is not defined by the load alone. Critical slowing down is its constructive counterpart. Because the return time to equilibrium diverges as the saddle-node is approached, and because this divergence reads in routine quantities, the rise in autocorrelation and variance of the fluctuations, an instrumented digester can signal the approach of its switch before it occurs. This opens a regime-aware operation and accounting: follow the indicator, reduce the load when it lights up, and attribute the credit only to an operation verified to stand at a distance from the fold. This is information no static indicator provides, and it has a direct practical value, since it distinguishes a robust credit from a credit that rests on a state the reactor might leave.

The interest of this signal lies in the fact that it requires no new measurement: it rests on the variables already monitored in operation. The state indicators of anaerobic digestion, volatile-fatty-acid concentration, ratio of acids to buffering capacity (FOS/TAC), dissolved hydrogen, are precisely those whose variance and autocorrelation carry the signature of critical slowing down (Boe et al., 2010). Concretely, detection is done on the time series: one computes, on a sliding window, the lag-one autocorrelation and the variance of the chosen indicator, and watches for their joint rise, rather than the crossing of a fixed threshold by the instantaneous value. The statistical methods for detecting these signals in real series are today standardized (Dakos et al., 2012), including the handling of the pitfalls, seasonal trend, false positives, which require crossing several indicators. The protocol that follows is sober: instrument the reactor, monitor the variance and autocorrelation of the acid indicators, reduce the load or reinforce the alkalinity as soon as these quantities rise jointly, and claim a climate credit only at a verified distance from the fold. The direct experimental mapping of this signature during a controlled load increase remains to be done; it would constitute the most direct validation of the present work.

5.4 Scope and limits

Several limits bound what we establish. Bistability and the saddle-node are properties of the AM2 model, whose parameters were identified on a real reactor (Bernard et al., 2001) and whose bistable structure was established analytically (Meadows et al., 2019); but the direct experimental mapping of the saddle-node and of critical slowing down on an instrumented digester, by monitoring the warning indicators over a controlled load increase, remains to be carried out, and it is this that would move this result from the status of a consequence of a validated model to that of a measured fact. This gap is, however, narrower than it appears. Critical slowing down has already been observed directly, with rising variance and autocorrelation in support, on a real microbial population approaching a tipping point through dilution (Dai et al., 2012), that is, in a system of the same nature and with the same type of control parameter as a digester. What remains to be shown is therefore not that the phenomenon exists, but that it reads specifically in the indicators of a digester; and a retrospective analysis of already-published operating series, where a reactor approached washout, could in principle establish it without a new experiment. The magnitude of the sign change of the climate balance depends on the assumed fixed operating load, which we declare without measuring; only the sign change is independent of this choice. Finally, we treat here only one process; we do not claim that every LCA process is bistable, but that the processes that are, and anaerobic digestion is a documented case, demand an accounting that takes it into account.

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6. Conclusion

Anaerobic digestion is a bistable process, an established fact of its dynamics. We have drawn from it a consequence for life cycle assessment: in the bistability window, the climate credit attributed to the process depends on the history of the reactor and not on the load alone, so that a single credit is ill-posed there, and an average between branches describes no real state. Advanced uncertainty treatment does not correct this flaw, because it is of form and not of precision. We have finally shown that critical slowing down provides a measurable early-warning signal of the switch, and that it opens a regime-aware accounting. For processes liable to bifurcate, LCA would gain by reporting the regime, and not only the quantity. A credit that depends on the history of the reactor is not a false number; it is a tacit bet on the branch the machine will occupy, and it is that bet, far more than the number, that an honest assessment must make visible.

References

- Batstone, D. J., Keller, J., Angelidaki, I., Kalyuzhnyi, S. V., Pavlostathis, S. G., Rozzi, A., Sanders, W. T. M., Siegrist, H., & Vavilin, V. A. (2002). The IWA Anaerobic Digestion Model No. 1 (ADM1). *Water Science and Technology*, 45(10), 65–73.

- Bernard, O., Hadj-Sadok, Z., Dochain, D., Genovesi, A., & Steyer, J.-P. (2001). Dynamical model development and parameter identification for an anaerobic wastewater treatment process. *Biotechnology and Bioengineering*, 75(4), 424–438.
- Boe, K., Batstone, D. J., Steyer, J.-P., & Angelidaki, I. (2010). State indicators for monitoring the anaerobic digestion process. *Water Research*, 44(20), 5973–5980.
- Dai, L., Vorselen, D., Korolev, K. S., & Gore, J. (2012). Generic indicators for loss of resilience before a tipping point leading to population collapse. *Science*, 336(6085), 1175–1177.
- Dakos, V., Carpenter, S. R., Brock, W. A., Ellison, A. M., Guttal, V., Ives, A. R., Kéfi, S., Livina, V., Seekell, D. A., van Nes, E. H., & Scheffer, M. (2012). Methods for detecting early warnings of critical transitions in time series. *PLoS ONE*, 7(7), e41010.
- Groen, E. A., Heijungs, R., Bokkers, E. A. M., & de Boer, I. J. M. (2014). Methods for uncertainty propagation in life cycle assessment. *Environmental Modelling & Software*, 62, 316–325.
- Heijungs, R., & Suh, S. (2002). *The Computational Structure of Life Cycle Assessment*. Springer.
- Li, L., He, Q., Wei, Y., He, Q., & Peng, X. (2017). Instability mechanisms and early warning indicators for mesophilic anaerobic digestion of vegetable waste. *Bioresource Technology*, 245(A), 90–97.
- McAvoy, S., Grant, T., Smith, C., & Bontinck, P.-A. (2021). Combining life cycle assessment and system dynamics to improve impact assessment: a systematic review. *Journal of Cleaner Production*, 315, 128060.
- Meadows, T., Weedermann, M., & Wolkowicz, G. S. K. (2019). Global analysis of a simplified model of anaerobic digestion and a new result for the chemostat. *SIAM Journal on Applied Mathematics*, 79(2), 668–689.
- Scheffer, M., Carpenter, S., Foley, J. A., Folke, C., & Walker, B. (2001). Catastrophic shifts in ecosystems. *Nature*, 413, 591–596.
- Scheffer, M., Bascompte, J., Brock, W. A., Brovkin, V., Carpenter, S. R., Dakos, V., Held, H., van Nes, E. H., Rietkerk, M., & Sugihara, G. (2009). Early-warning signals for critical transitions. *Nature*, 461, 53–59.
- Strogatz, S. H. (2018). *Nonlinear Dynamics and Chaos* (2nd ed.). CRC Press.
- Wu, D., Li, L., Zhao, X., Peng, Y., Yang, P., & Peng, X. (2014). Early warning indicators for monitoring the process failure of anaerobic digestion system of food waste. *Bioresource Technology*, 171, 308–316.

- EN 15804:2012+A2:2019. *Sustainability of construction works, environmental product declarations.*

Supplement

Reproducible code, parameter table and robustness sweeps: Zenodo deposit (DOI at the first stable version). The two figures are regenerated by the computation scripts provided; the robustness sweeps (notably on `KI2`) are included.